On the properties of linear supply functions in oligopoly*

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Abstract

In this note we revisit the result by Menezes and Quiggin (2012), showing that under linear supply function competition, the same Nash equilibrium results when firms choose slopes or intercepts of their supply functions. This is because the first order conditions emerging in the two strategy spaces are not linearly independent.

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1 Introduction

After the seminal paper by Klemperer and Meyer (1989), the literature on supply functions has been growing in several directions. This is no surprise, because supply function competition represents an interesting option when modelling oligopolistic interaction. However, the complexity of the Klemperer and Meyer (1989) setting has stimulated approaches designed to make their original model more tractable.

More precisely, we have witnessed a special focus on linear supply functions. One of the models proposed in this vein is in Menezes and Quiggin (2012), where firms choose the intercept of their linear supply functions, while the slope is treated as a parameter, in such a way that, if the slope is nil, the equilibrium outcome replicates Cournot, whereas if the slope is infinitely high, it replicates Bertrand or perfect competition.

Menezes and Quiggin's (2012) formulation, however, allows one to treat also the slope as a strategic variable. This is what we do in this paper. We show that the resulting first order conditions in the extended strategy space formed by both intercepts and slopes are not linearly independent and the Nash equilibrium generated by choosing either one is the same.

We set up the model in section 2. Section 3 contains our results, and section 4 concludes by underlying some extensions.

2 The model

Consider an oligopoly formed by n identical firms selling a homogeneous good whose market demand function is $p = a - Q$, where p is market price, $a > 0$ is a parameter measuring market size (which is normalised to one in

Menezes and Quiggin (2012)) and industry output $Q = \sum_{i=1}^{n} q_i$ is the sum of all individual outputs q_i. Firm i's cost function is $C_i = cq_i$ where parameter $c \in (0, a)$. Menezes and Quiggin (2012) specify the individual firm's supply function as $q_i = \alpha_i - c/n + \beta\,(p - c)$, where intercept $\alpha_i > 0$ is the strategic variable, while slope $\beta \geq 0$ is a parameter. It is worth noting that this supply function is increasing in the mark-up; the intercept is normalised the characterisation of equilibrium.

In view of Ciarreta and Gutierrez-Hita (2006), where the supply function is $q_i = \beta_i p$ and the slope is the strategic variable, we extend Menezes and Quiggin's (2012) approach by stipulating that the supply function is as follows:

$$q_i = \alpha_i - \frac{c}{n} + \beta_i\,(p - c) \tag{1}$$

where both the intercept $\alpha_i > 0$ and the slope $\beta_i \geq 0$ are strategic variables.

Solving the *ex ante* market-clearing condition, we get

$$p = \frac{a - \sum_{i=1}^{n} \alpha_i + c\,(1 + \sum_{i=1}^{n} \beta_i)}{1 + \sum_{i=1}^{n} \beta_i} \tag{2}$$

so that the individual profit function is

$$\begin{gathered} \pi_i = (p - c)\, q_i = \\ \left[\frac{a - \sum_{i=1}^{n} \alpha_i + c\,(1 + \sum_{i=1}^{n} \beta_i)}{1 + \sum_{i=1}^{n} \beta_i} - c\right]\left(\alpha_i - \frac{c}{n}\right) + \\ \beta_i \left[\frac{a - \sum_{i=1}^{n} \alpha_i + c\,(1 + \sum_{i=1}^{n} \beta_i)}{1 + \sum_{i=1}^{n} \beta_i} - c\right]^2 \end{gathered} \tag{3}$$

3 Results

While Menezes and Quiggin (2012, p. 713) assume $\beta_i = \beta$ for all $i = 1, 2, 3, ..., n$ and take first order conditions (FOCs) w.r.t. α_i's, we calculate the FOCs w.r.t. both α_i's and β_i's:

$$\frac{\partial \pi_i}{\partial \alpha_i} = \frac{c\Phi - n\left[\left(2\alpha_i + \sum_{j \neq i} \alpha_j\right)\Theta + a\left(\beta_i - \Theta\right)\right]}{n\Phi^2} = 0 \tag{4}$$

$$\frac{\partial \pi_i}{\partial \beta_i} = \frac{\left(a - \sum_{i=1}^{n} \alpha_i\right)\left[c\Phi - n\left(\left(2\alpha_i + \sum_{j \neq i} \alpha_j\right)\Theta + a\left(\beta_i - \Theta\right)\right)\right]}{n\Phi^3} = 0 \tag{5}$$

where $\Phi \equiv 1 + \sum_{i=1}^{n} \beta_i$ and $\Theta \equiv 1 + \sum_{j \neq i} \beta_j$. Looking at the system (4-5), it appears that

$$\frac{\partial \pi_i}{\partial \beta_i} = \frac{\partial \pi_i}{\partial \alpha_i} \cdot \frac{a - \sum_{i=1}^{n} \alpha_i}{\Phi} \tag{6}$$

which implies the following:

Lemma 1 *If the strategy space of the game in supply functions includes both the slope and the intercept of each firm's supply, strategic variables are not linearly independent.*

The above Lemma immediately implies a relevant consequence:

Proposition 2 *Under linear supply competition, the Nash equilibrium is independent of the combination of intercepts and slopes chosen by firms.*

That is to say, the Nash equilibrium of this game can be equivalently obtained by (i) posing $\beta_i = \beta$ in (4) and then solving for α_i or (ii) posing $\alpha_i = \alpha$ in (5) and then solving for β_i. This entails that firm i may not even know (and in fact, doesn't care to know) whether any rival firm j is using (4) or (5).

It is a matter of simple algebra to show that the two methods indeed lead to the same outcome. Procedure (i) yields the same Nash equilibrium solution as in Menezes and Quiggin (2012, eq. (5), p. 713):

$$\alpha^* = \frac{c\left(1 + n\beta\right) + an\left[1 + \beta\left(n - 2\right)\right]}{n\left[n + 1 + n\beta\left(n - 1\right)\right]} \tag{7}$$

Procedure (ii) yields

$$\beta^* = \frac{c + n\left[a - \alpha\left(n+1\right)\right]}{n\left[n\alpha\left(n-1\right) - a\left(n-2\right) - c\right]} \tag{8}$$

and then it is easily verified that α^* and β^* are invertible, with $\alpha^* = \left(\beta^*\right)^{-1}$. Using α^*, the equilibrium price p^* is as in Menezes and Quiggin (2012, eq. (6), p. 173), with

$$p^*|_{\beta=0} = \frac{a+cn}{n+1}\ ;\ \lim_{\beta\to\infty} p^* = c \tag{9}$$

reproducing the special cases of Cournot and Bertrand (or perfect competition), respectively. If one uses instead β^*, the resulting price is

$$p^* = \frac{c + n\alpha\left(n-1\right) - a\left(n-2\right)}{2} \tag{10}$$

with

$$p^* = \frac{a+cn}{n+1} \text{ at } \alpha = \frac{an+c}{n\left(n+1\right)} = \alpha^*|_{\beta=0} \tag{11}$$

and

$$p^* = c \text{ at } \alpha = \frac{a\left(n-2\right)+c}{n\left(n-1\right)} = \lim_{\beta\to\infty}\alpha^* \tag{12}$$

Then, the equivalence between the two above procedures is complete.

The intuition behind this result can be explained as follows. Under the market clearing condition, each firm operates along the residual demand curve left by the supply chosen by rivals. Hence, since supply functions are linear, it's indifferent to determine the optimal supply by choosing the intercept or the slope. To see this, take the standpoint of firm i, whose supply function is defined in (1), and suppose firm i takes the rivals' collective supply $Q_{-i} = \sum_{j\neq i} q_j$ as given. Hence, the residual demand faced by firm i is $D_R = a - p - Q_{-i}$, which, by the market clearing condition, must be equal to q_i. Imposing $Q_{-i} = \left(n-1\right)q_j$ and solving $D_R = q_i$ w.r.t. price, we obtain

$$p = \frac{n\left[a - \alpha_i - \left(n-1\right)q_j\right] + c\left(n\beta_i + 1\right)}{n\left(1+\beta_i\right)} \tag{13}$$

Firm i's profits are again defined as $\pi_i = (p-c)\, q_i$. Taking FOCs w.r.t. α_i and β_i, we get

$$\frac{\partial \pi_i}{\partial \alpha_i} = \frac{n\left[a\left(1-\beta_i\right) - 2\alpha_i + \left(\beta_i - 1\right)(n-1)\, q_j\right] + c\left[2 + n\left(\beta_i - 1\right)\right]}{n\left(1+\beta_i\right)^2} = 0 \tag{14}$$

$$\frac{\partial \pi_i}{\partial \beta_i} = -\frac{c\,(n-1) + n\left[\alpha_i - a + (n-1)\, q_j\right]}{n\left(1+\beta_i\right)} \cdot \frac{\partial \pi_i}{\partial \alpha_i} = 0 \tag{15}$$

which jointly imply that the above intuitive explanation is indeed correct.

4 Extensions

The result stated in Proposition 1 extends to a more general setup in which the cost function includes a quadratic component, becoming $C_i = q_i\left(c + bq_i/2\right)$, as in Ciarreta and Gutierrez-Hita (2006).[1] In this case, one may simply rewrite the individual supply function as

$$q_i = \alpha_i - \frac{\partial C_i / \partial q_i}{n} + \beta_i\,(p-c) \tag{16}$$

and then proceed as follows. Taking FOCs w.r.t. α_i and β_i and imposing the symmetry conditions $\alpha_j = \alpha_i$ and $\beta_j = \beta_i$ for all $j \neq i$, we obtain the following expressions:

$$\begin{gathered}\frac{\partial \pi_i}{\partial \alpha_i} = \frac{n}{(b+n)\left[b + n\left(n\beta_i + 1\right)\right]^2}\left[(b+n)\left(c + a\,(b+n) - n\,(n+1+b)\,\alpha_i\right) - \right. \\ \left. n\,(b+n)\left(n\,(n-1)\,\alpha_i - (1+b)\,c - a\,(n-2-b)\right)\beta_i - b\,(a-c)\,(n-1)\,n^2\beta_i^2\right] = 0\end{gathered} \tag{17}$$

$$\frac{\partial \pi_i}{\partial \beta_i} = -\frac{n\left[a\,(b+n) - bc - n^2\alpha_i\right]}{(b+n)\left[b + n\left(n\beta_i + 1\right)\right]^3}\left[(b+n)\left(c + a\,(b+n) - n\,(n+1+b)\,\alpha_i\right) - \right. \tag{18}$$

[1] In Klemperer and Meyer (1989), the marginal cost is actually assumed to be non-negative.

$$n\,(b+n)\,(n\,(n-1)\,\alpha_i - (1+b)\,c - a\,(n-2-b))\,\beta_i - b\,(a-c)\,(n-1)\,n^2\beta_i^2\big] = 0$$

so that we may write

$$\frac{\partial \pi_i}{\partial \beta_i} = -\frac{\partial \pi_i}{\partial \alpha_i} \cdot \frac{a\,(b+n) - bc - n^2\alpha_i}{[b + n\,(n\beta_i + 1)]} \tag{19}$$

which proves that (17) and (18) are not linearly independent. Consequently, it can also be shown that exactly the same equilibrium outcome arises if the game takes a two-stage structure in which the first stage takes place in the α-space and the second stage in the β-space (or the opposite).

Finally, one may wonder about the specific normalisation adopted by Menezes and Quiggin (2012). It can be easily shown that our result does not depend on it, by reconstructing the foregoing argument under the assumption that the supply function is written as $q_i = \alpha_i + \beta_i p$.

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